A New Integral Formulation for Eddy Currents Computation in Thin Conductive Shells

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Abstract — In order to compute eddy currents in thin conductive non-magnetic shells, an integral formulation is proposed. Based on a shell element formulation, it is general and enables to the modeling of various problems whatever their skin-depth.

I. Introduction

The problem of eddy currents computation in thin shells for the case of a skin depth δ much greater than the thickness *e* ($\delta >> e$) has been treated by many authors [1]-[4]. In this case, the eddy currents distribution is supposed to be uniform across the thickness and surfaces elements are used. Such formulations are currently well-known and have shown good accuracies with a few numbers of elements in comparison with finite element methods where the air region needs to be meshed.

However, the computation of eddy currents in thin shells in the general case ($\delta << e$ or $\delta \approx e$ for instance) is still a difficulty and has only been studied by few authors. In [5], a shell element formulation has been proposed. Based on a pseudo-analytical solution (solution of the 1D problem) and a nodal approximation of the scalar magnetic potential, it enables to take into account the field variation across the thickness of the shell (i.e. the skin depth) for 3D geometry with a quite good accuracy. In [6], shell element formulation has been introduced in finite elements method.

This paper presents an integral formulation which adapts to modeling the thin shell conductive non-magnetic in the case general ($\delta >> e$ or $\delta \approx e$ or $\delta << e$). By using the simple surface meshing of the shell, the number of unknown is thus reduced considerably.

II. Formulation

A. Equations for the shell

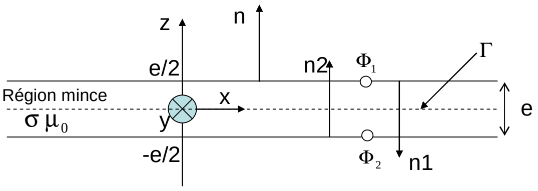

Fig.1 Thin conductive region

We consider that a small skin depth δ is associated to the non-magnetic shell with a thickness e (Fig.1). The field variation of the tangential component across thickness of shell is given by the analytical solution of the problem for an infinite plane [5], [6]:

$$\mathbf{H}_s(z) = (1/\mathrm{sh}(ae))\left[\mathbf{H}_{1s}\,\mathrm{sh}(ae/2+az) + \mathbf{H}_{2s}\,\mathrm{sh}(ae/2-az)\right] \quad (1)$$

where $a = (1+j)/\delta$, $\mathbf{H}_{1s}$ and $\mathbf{H}_{2s}$ are the field tangential values on both sides of the shell. Applying Galerkin method on the Maxell-Faraday equations for side "1" of the shell on a real 3D surface Γ, we get:

$$\int_\Gamma \mathrm{grad}_s w(\alpha\mathbf{H}_{1s} - \beta\mathbf{H}_{2s})d\Gamma + j\omega\mu_0 \int_\Gamma w\,\mathbf{H}_1\mathbf{n}_1 d\Gamma = 0 \quad (2)$$

where $\alpha = a/(\sigma\mathrm{th}(ae))$ $\beta = a/(\sigma\mathrm{sh}(ae))$; w is a set of nodal surface weighting functions; $\mathbf{n}_1$ is the normal vector corresponding the side "1" of the shell.

The other equation corresponding to the other side of the shell is obtained when indices "1" and "2" are exchanged:

$$\int_\Gamma \mathrm{grad}_s\, w(\alpha\mathbf{H}_{2s} - \beta\mathbf{H}_{1s})d\Gamma + j\omega\mu_0 \int_\Gamma w\,\mathbf{H}_2\mathbf{n}_2 d\Gamma = 0 \quad (3)$$

where $\mathbf{n}_2$ is the normal vector corresponding the side "2" of the shell.

B. Integral formulation:

In this part, we consider the side "1" of the shell. The volume courant density being tangential, it can be expressed by:

$$\mathbf{J} = \mathrm{curl}\mathbf{H} = -\mathbf{n}_1 \times \partial\mathbf{H}_s(z)/\partial z\,. \quad (4)$$

Thus, by deriving (1), we get:

$$\mathbf{J}(z) = (-a/\mathrm{sh}(ae))\mathbf{n}_1 \times \left[\mathbf{H}_{1s}\mathrm{ch}(ae/2+az) - \mathbf{H}_{2s}\mathrm{ch}(ae/2-az)\right]\,. \quad (5)$$

Let us now consider that the shell is placed in an inductor field $\mathbf{H}_0$. The total magnetic field H_1 is the sum of H_{01} and H_{1r}, the reaction of the eddy current in thin shell:

$$\mathbf{H}_1 = \mathbf{H}_{01} + (1/4\pi)\int_\Gamma \int_{-e/2}^{e/2} \mathbf{J}(z) \times (\mathbf{r}/r^3)dz\,d\Gamma \quad (6)$$

where **r** is the vector linking the integration point to the point where the field is expressed (Fig.2). Additional expressions can be given:

$$\mathbf{r} = \mathbf{r}_s - (e/2 - z).\mathbf{n}_1 \text{ and } r^3 = \left(r_s^2 + (e/2-z)^2\right)^{3/2}. \quad (7)$$

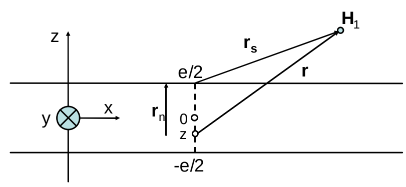

Fig.2 Field magnetic on the side "1"created by eddy current

Using (5), (6) and (7), we obtain:

$$\mathbf{H}_1 = \mathbf{H}_{01} + (1/4\pi)\int_\Gamma (\mathbf{R}_1.\mathbf{H}_{1s} - \mathbf{R}_2.\mathbf{H}_{2s})d\Gamma \quad (8)$$

with:

$$\mathbf{R}_1 = -\frac{a}{\mathrm{sh}(ae)} \int_{-e/2}^{e/2} \frac{\mathrm{ch}(ae/2 + az)\mathbf{n}_1 \times (\mathbf{r}_s - (e/2 - z).\mathbf{n}_1)}{\left(r_s^2 + (e/2 - z)^2\right)^{3/2}} dz \quad (9)$$

$$\mathbf{R}_2 = -\frac{a}{\mathrm{sh}(ae)} \int_{-e/2}^{e/2} \frac{\mathrm{ch}(ae/2 - az)\mathbf{n}_1 \times (\mathbf{r}_s - (e/2 - z).\mathbf{n}_1)}{\left(r_s^2 + (e/2 - z)^2\right)^{3/2}} dz \,. \quad (10)$$

Using (8), equation (2) becomes:

$$\int_\Gamma \mathrm{grad}_s\, w(\alpha \mathbf{H}_{1s} - \beta \mathbf{H}_{2s}) d\Gamma$$
$$+ j\omega\mu_0 \int_\Gamma w \left(\mathbf{H}_{01} + \frac{1}{4\pi} \int_\Gamma (\mathbf{R}_1 \mathbf{H}_{1s} - \mathbf{R}_2 \mathbf{H}_{2s}) d\Gamma \right) . \mathbf{n}_1 d\Gamma = 0 \quad . (11)$$

Let us now introduce the reduced magnetic scalar potential. Tangential magnetic fields on both sides of shell are written:

$$\mathbf{H}_{1s} = \mathbf{H}_{0s} - \mathrm{grad}_s\, \phi_1 \quad (12)$$
$$\mathbf{H}_{2s} = \mathbf{H}_{0s} - \mathrm{grad}_s\, \phi_2 \quad (13)$$

where $\mathbf{H}_{0s}$ is the tangential inductor field. ϕ_1 and ϕ_2 stand for magnetic scalar potentials on both sides.

Using (11), (12) and (13), we obtain the final formulation corresponding to side "1" of the shell:

$$\begin{aligned} &-\alpha \int_\Gamma \mathrm{grad}_s\, w.\mathrm{grad}_s \phi_1 d\Gamma + \beta \int_\Gamma \mathrm{grad}_s\, w.\mathrm{grad}_s \phi_2 d\Gamma \\ &+ (j\omega\mu_0 / 4\pi) \int_\Gamma w. \left(\int_\Gamma -\mathbf{R}_1\, \mathrm{grad}_s\, \phi_1 d\Gamma + \int_\Gamma \mathbf{R}_2\, \mathrm{grad}_s\, \phi_2 d\Gamma \right) d\Gamma \\ &= -(j\omega\mu_0 / 4\pi) \int_\Gamma w. \left[\int_\Gamma (\mathbf{R}_1 - \mathbf{R}_2) \mathbf{H}_{0s}\, d\Gamma \right] d\Gamma \\ &+ (-\alpha + \beta) \int_\Gamma \mathrm{grad}_s\, w.\mathbf{H}_{0s} d\Gamma - j\omega\mu_0 \int_\Gamma w \mathbf{H}_0 \mathbf{n}_1 d\Gamma \end{aligned} \quad (14)$$

The equation on side "2" of shell is obtained by permuting subscripts "1" and "2". These equations have to be discretized. The easiest way is to mesh the surface Γ into n triangular elements associated with a constant tangential component of eddy current (1-order shape functions for the potential). The global square matrix system obtained has $2p$ equations (two complex magnetic scalar potentials per node; the mesh is being composed of p nodes).

III. Numerical Example

In this part, we consider two numerical examples. For both, our formulation is compared with results given by 2D FEM provided in Flux software [7]. In order to test them, we focus on the computed Joule losses at different frequencies.

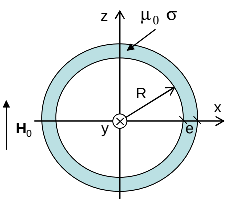

Fig.3 Hollow sphere

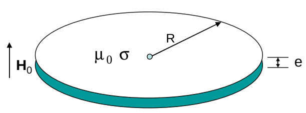

Fig.4 Thin conductive disk

In the first example, a conductive hollow sphere (R=0.1m, e=2E-3m, σ=6E7 S/m) is placed in a uniform axial magnetic field H_0= [0 0 1] (A/m) (Fig.3). The second test case is a thin conductive disk (R=1m, e=50E-3m, σ=6E7 S/m). This disk is placed in a uniform magnetic field H_0= [0 0 1] (A/m) (Fig.4).

Table I and Table II show the Joule losses values computed by our method and 2D FEM method in the sphere and the disk, respectively. We can see a small differences between both computed values. Of course, the differences are larger with small values of δ compared to e. So the surface mesh has to be refined in order to take into account the rapid changing of current densities.

TABLE I
JOULE LOSSES OF THE HOLLOW SPHERE

e/δ	Loss Joule (W) computed by our method	Loss Joule (W) computed by FEM 2D	Diff.
0.22	1.16E-6	1.14E-6	1.74%
0.90	1.75E-6	1.71E-6	2.34%
2.00	2.79E-6	2.90E-6	3.71%

TABLE II
JOULE LOSSES OF THE DISK

e/δ	Loss Joule (W) computed by our method	Loss Joule (W) computed by FEM 2D	Diff.
0.08	7.31E-9	7.33E-9	0.02%
0.77	6.38E-6	6.32E-6	1.00%
1.33	9.22E-6	9.54E-6	3.30%

IV. Conclusion

In this paper, we have presented an integral formulation using shell elements in order to model thin conductive non – magnetic regions. The formulation is general and various skin effect across thickness ($\delta >> e$ or $\delta \approx e$ or $\delta << e$) are taken into account. Moreover, a combination with MoM and PEEC method [8] are being investigated in order to model industrial device. In further word, formulations enabling thin conductive magnetic shell will be researched.

V. References

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